A R C H I W U M I N S T Y T U T U I N Ż Y N I E R I I L Ą D O W E J
Nr 12 ARCHIVES OF INSTITUTE OF CIVIL ENGINEERING 2012

COMPUTER SOFTWARE FOR CALCULATING INTERNAL AND EXTERNAL FORCES IN CORRUGATED CULVERTS ON THE BASIS OF MEASURED STRAINS

Leszek KORUSIEWICZ*
*) PhD Mech. E., Wrocław University of Technology, Poland

Abstract

This paper presents computer software for calculating the internal bending moments and normal and shearing forces in corrugated culverts as well as external normal and shearing forces acting via the ground on the culvert surface. Two methods (interpolation and extrapolation) are used to compute the quantities. The basic input data include the material and geometrical characteristics of the tested culvert and the circumferential stresses calculated on the basis of the measured strains. The main procedures used in the software are described. The results of computations performed by the software for a theoretical case are presented.

Key words: computer software, corrugated culverts, full-size tests, stresses, internal and external forces

1. INTRODUCTION

Numerous full-scale tests in the field and under fully controlled laboratory conditions have been carried out to determine long-term performance and load-bearing capacity of corrugated culverts [1–6]. Experimental results give an invaluable insight into the actual behaviour of such structures.

This paper presents dedicated computer software for calculating internal normal and shearing forces and bending moments in corrugated culverts and external normal and shearing forces acting via the surrounding ground onculvert surface.

2. DESCRIPTION OF SOFTWARE

The software, under the working name of TUNEL2, computes load intensities on the surface and generalized cross-sectional forces in the lining of a tunnel (culvert) on the basis of circumferential stresses measured in two layers oftunnel lining. Data from measuring points (in cross sections) uniformly distributed on the lining's average contour are input into the software. The software comprises two subprograms called TUNEL2-D and TUNEL2-O.

2.1 Program TUNEL2-D

This program is used to create a data file for program TUNEL2-O. The input data are:

– task identifier,
– number of strain gauge pairs (the number of measuring points) – *lpt*,
– Young modulus of the culvert material – *E*,
– lining membrane stiffness in the circumferential direction – D_φ,
– lining flexural rigidity in the circumferential direction – K_φ,
– distance of measuring points D_i from the middle layer (the distance is positive for points D_i within the average contour of the lining) – g_{Di},
– distance of measuring points G_i from the middle layer (the distance is negative for points G_i outside the average contour of the lining) – g_{Gi} ($g_{Di} = g_D$ = const and $g_{Gi} = g_G$ = const are assumed in the program),
– length of the average lining contour – *SKOT* (the average lining contour should be understood as the arc of a curve, being the geometric locus of the centres of gravity of the considered tunnel lining segment cross sections).

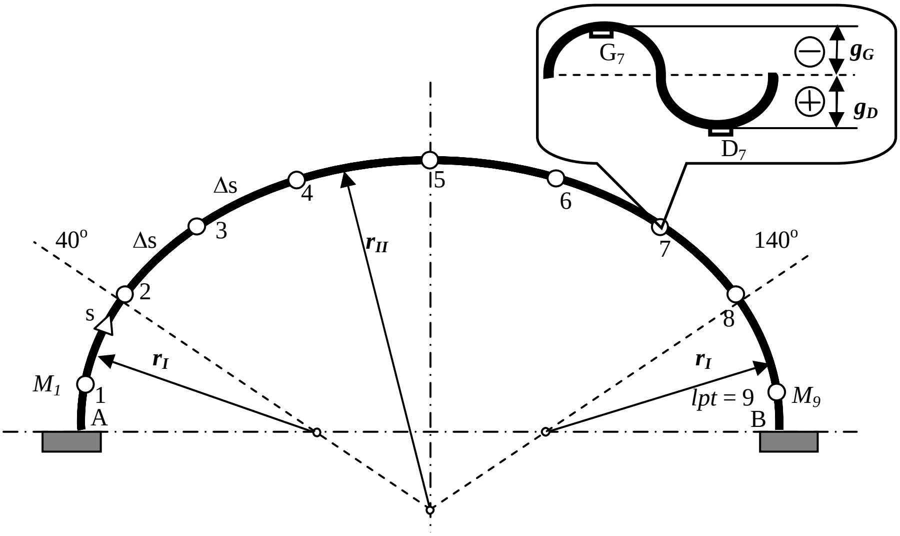
For the next control (measuring) cross sections bearing numbers $i = 1, 2,\ldots, lpt$ and ordered according to increasing length s_i the following quantities are entered:

– distance of a given measuring point from the adopted starting point, measured along the average lining contour – s_i,
– radius of curvature of the average lining contour – r_i,
– circumferential stress in points D_i and G_i – σ_{Di} and σ_{Gi}.

Moreover, the program requires that it be specified whether the average lining contour is an open or closed curve and whether the radius of curvature along the whole length of the average contour is intervalwise constant. If so, the number of intervals in which the radius of curvature is constant needs to be input.

Into the program, data can be fed in the interactive mode and saved to a file with a name given by the user or read in from a file indicated by the user.

The quantities used in program TUNEL2-D are explained in Figure 1.



SKOT – length of arc $\overset{\frown}{M_1 M_9}$

Figure 1.Diagram of culvert structure

Circumferential stresses σ_{Di} and σ_{Gi} are calculated on the basis of the strains measured in the measuring points. If bidirectional strain rosettes are used (Fig. 2), the circumferential strains are calculated from the following relation:

$$\sigma_D = \frac{E}{1-\nu^2}\left(\varepsilon_{D_C} + \nu\varepsilon_{D_L}\right),\ \ \sigma_G = \frac{E}{1-\nu^2}\left(\varepsilon_{G_C} + \nu\varepsilon_{G_L}\right) \tag{2.1}$$

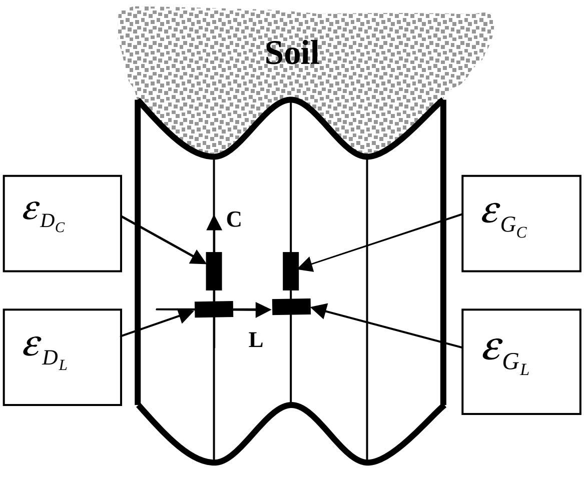


Figure 2. Location of strain gauges on inner side of culvert lining

Membrane stiffness D_φ and flexural rigidity K_φ for corrugated culverts are expressed by the equations:

$$K_\varphi = E\, I_f\,, \quad D_\varphi = E\, A_f\,, \tag{2.2}$$

where A_f and I_f stand for respectively the area and moment of inertia of the cross section (fold) per width unit.

2.2 Program TUNEL2-O

This computational program uses the data saved in program TUNEL2-D. The internal forces in the culvert shell and the external forces acting on the culvert shell from the surrounding ground (Fig. 3) are computed. The results of the computations are displayed in the form of tables and diagrams on the computer screen.

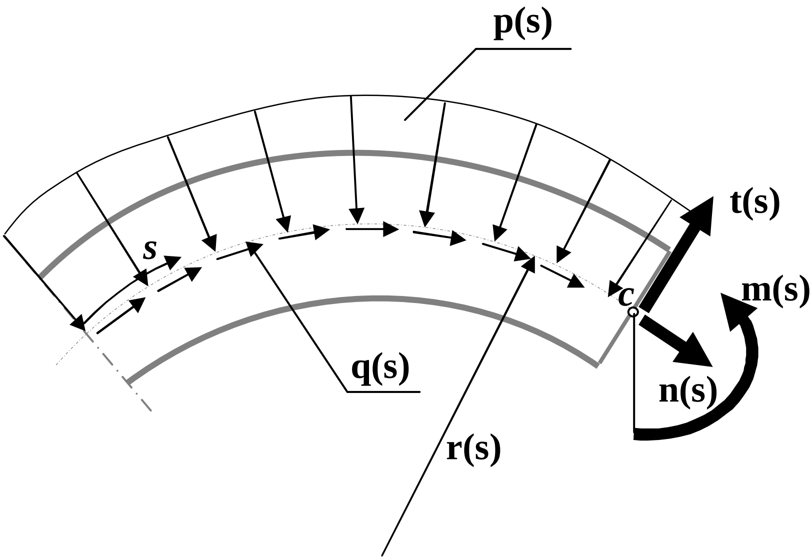


Figure 3. Internal and external forces acting on culvert structure

The symbols in Figure 3 stand for: $p(s)$ – the normal stress per middle lining layer unit area; $q(s)$ – the static load per middle lining layer unit area; $m(s)$, $n(s)$ and $t(s)$ – the bending moment, the normal force and the shearing force in the cross section of the lining per cross sectional unit width.

First the program calculates bending moments m_i and normal forces n_i ($i = 1, 2, \ldots, lpt$) in the control (measuring) cross sections from the formulas:

$$m_i = \frac{K_\varphi}{E(g_{Di} - g_{Gi})}(\sigma_{D_i} - \sigma_{Gi})$$
$$n_i = \frac{D_\varphi}{E(g_{Di} - g_{Gi})}(g_D\,\sigma_{Gi} - g_G\,\sigma_{Di}) \tag{2.3}$$

Then the coefficients of the Fourier-Bessel series of functions $m(s)$ and $n(s)$ are calculated and taking into account the statics conditions:

$$t = -\frac{\mathrm{d}m}{\mathrm{d}s}, \tag{2.4}$$

$$p = \frac{\mathrm{d}^2 m}{\mathrm{d}s^2} + \frac{n}{r}, \tag{2.5}$$

$$q = -\frac{\mathrm{d}n}{\mathrm{d}s} - \frac{t}{r}, \tag{2.6}$$

the coefficients of the Fourier-Bessel series of functions $t(s)$, $p(s)$ and $q(s)$ are calculated. The user can choose the trigonometric interpolation method or the trigonometric approximation method. The program correctly solves the interpolation problem only when the distances between the control cross sections are constant Δs = const.

Two approximation selection methods: a standard method and a statistical method are available in the program. In both cases, the so-called approximation defect is calculated for successive tentative values of the number of harmonic components (LSH) in expansions $m(s)$ and $n(s)$ into Fourier-Bessel series. As the proper LSH value the user should select the value above which the approximation defects no longer significantly decrease. In the standard method one uses an approximation defect defined as the ratio of the sum square of the measured ordinate deviations from the calculated ordinates in the control cross sections to the sum square of the ordinates measured in the control cross sections. In the statistical method the approximation defect is an approximation error variance defined in the monograph by A. Ralston [7].

The user can select from three methods of calculating the derivatives of functions in the nodes, mainly the method of Fourier series differentiation, the finite difference method with the five-point scheme and the finite difference method with the three-point scheme.

The calculation results can be saved to a file with a name given by the user as “Results table I” in which values m_i and n_i calculated from formulas (2.3) are stored and as “Results table II” which for j uniformly distributed cross sections (j – a number selected by the user, lower than 362) stores values $m(s)$ and $n(s)$

determined by the trigonometric interpolation method together with values $m(s)$, $n(s)$ and $t(s)$ determined (depending on the user's needs) by the trigonometric approximation or interpolation method and values $p(s)$ and $q(s)$ determined by the trigonometric approximation method or the trigonometric interpolation method.

3. VERIFICATION OF PROGRAM

The following case was used to verify program TUNEL2. For the culvert shown in Figure 1 the following parameters values were adopted: length of arc AB = 488.69, length of average contour (length of arc M_1M_2) *SKOT* = 453.79, number of strain gauge pairs *lpt* = 9, Young modulus $E = 1$, membrane stiffness $D_\varphi = 1$, flexural rigidity $K_\varphi = 17$, distance $g_D = 6$, distance $g_G = -6$, intervalwise constant radius of curvature, number of intervals = 3:

from s	to s	radius
17.45	69.81	$r_I = 100$
69.81	418.88	$r_{II} = 200$
418.88	471.2G	$r_I = 100$

When preparing the data, the following hypothetical functions were assumed:

$$\sigma_D(s) = -2 - \cos\xi^\circ,$$
$$\sigma_G(s) = -2 + \cos\xi^\circ, \tag{3.1}$$

where $\xi^\circ = 360^\circ s/s_0$, $s_0 = 488.69$ is the length of arc AB and s is a coordinate measured along arc from point A. For these functions $m(s)$ and $n(s)$ were calculated from relations (2.3) and then $t(s)$, $p(s)$ and $q(s)$ were calculated from (2.4), (2.5) and (2.6). The appropriate relations assume this form:

$$m = -2.83\cos\xi^\circ,$$
$$n = -2 \tag{3.2}$$

$$t = -0.036\sin\xi^\circ,$$
$$p = -\frac{2}{r} + 0.00047\cos\xi^\circ, \tag{3.3}$$
$$q = \frac{0.036}{r}\sin\xi^\circ.$$

As the measuring point stresses to be input into the program the following discrete values: $\sigma_D(s)$ and $\sigma_G(s)$ for $s = s_i$, $i = 1, 2, \ldots, 8, 9$ were adopted:

Measuring point i	s_i	σ_{Di}	σ_{Gi}
1	17.45	−2.975	−1.025
2	74.17	−2.579	−1.421
3	130.90	−1.888	−2.112
4	187.62	−1.254	−2.746
5	244.35	−1.000	−3.000
6	301.07	−1.254	−2.746
7	357.79	−1.888	−2.112
8	414.52	−2.579	−1.421
9	471.24	−2.975	−1.025

The results of the analytical computations are shown in Table 1.

Table 1. Results of analytical computations

Measuring point i	m_i	n_i	t_i	p_i	q_i
1	−2.76	−2.00	− 0.0080	− 0.0195	0.000080
2	−1.64	−2.00	− 0.0294	− 0.0097	0.000147
3	0.32	−2.00	− 0.0358	− 0.0101	0.000179
4	2.11	−2.00	− 0.0240	− 0.0104	0.000120
5	2.83	−2.00	0.0000	− 0.0105	0.000000
6	2.11	−2.00	0.0240	− 0.0104	− 0.000120
7	0.32	−2.00	0.0358	− 0.0101	− 0.000179
8	−1.64	−2.00	0.0294	− 0.0097	− 0.000147
9	−2.76	−2.00	0.0080	− 0.0195	− 0.000080

The results of the computations performed by program TUNEL2 (the interpolation method, the derivatives calculated by the finite difference method with the five-point scheme) are shown in Table 2.

Table 2. Results of computations performed by program TUNEL2

Measuring point i	m_i	n_i	t_i	p_i	q_i
1	−2.76	−2.00	− 0.0124	− 0.0197	0.000124
2	−1.64	−2.00	− 0.0286	− 0.0097	0.000143
3	0.32	−2.00	− 0.0359	− 0.0101	0.000180
4	2.11	−2.00	− 0.0241	− 0.0103	0.000120
5	2.83	−2.00	0.0000	− 0.0105	0.000000
6	2.11	−2.00	0.0241	− 0.0103	− 0.000120
7	0.32	−2.00	0.0359	− 0.0101	− 0.000180
8	−1.64	−2.00	0.0286	− 0.0097	− 0,000143
9	−2.76	−2.00	0.0124	− 0.0198	− 0.000124

A comparison of the content of Tables 1 and 2 shows that the theoretical results and the ones computed by program TUNEL2 are in very good agreement.

4. ACKNOWLEDGEMENTS

The author hereby thanks Mr. B. Okołów for his contribution to the development of the computer software and for his comments concerning this paper.

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Streszczenie

Niniejsza praca przedstawia oprogramowanie komputerowe do obliczania wewnętrznych momentów zginających oraz normalnych i tnących poprzecznych sił w przepustach z blachy falistej, a także zewnętrznych normalnych i tnących poprzecznych sił działających poprzez grunt na powierzchnię przepustu. Do obliczania wartości używane są dwie metody (interpolacja i ekstrapolacja). Podstawowe dane wejściowe obejmują charakterystykę materiałową i geometryczną badanego przepustu oraz naprężenia obwodowe obliczone na podstawie zmierzonych odkształceń. Opisano główne procedury stosowane w oprogramowaniu. Przedstawiono rezultaty obliczeń wykonanych przez oprogramowanie dla teoretycznego przykładu.

Słowa kluczowe: oprogramowanie komputerowe, przepusty z blachy falistej, próby pełnowymiarowe, naprężenia, siły wewnętrzne i zewnętrzne