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A SIMPLIFIED SOLUTION FOR THE DESIGN OF SLOTTED JOINT CORRUGATED STEEL STRUCTURAL PLATE PIPE

Kevin WHITE[*], Shad SARGAND[**]
[*]) Water Resources Group Manager, E.L. Robinson Engineering, 1801 Watermark Drive, STE 310, Columbus, OH, 43215, USA
[**]) Russ Professor of Civil Engineering, Ohio University, 145 Stocker Center, Athens, OH 45701, USA

Abstract

Corrugated steel pipes fabricated with slotted joints allow mating plates to slip over one another by a predetermined amount. This ring shortening has several implications for the installed pipe. The primary benefit is a reduction in the soil pressure acting on the pipe and a resultant reduction in the thrust in the pipe wall.

This paper investigated the response of the installed slotted joint, structural plate corrugated steel pipe by means of the finite element method. A simplified solution for the design of slotted joint corrugated steel structural plate pipe was developed and compared to the FEM results.

Key words: Corrugated steel pipe, soil-structure interaction, slotted-joint culvert

1. INTRODUCTION

Corrugated steel pipe is an attractive option for a multitude of culvert applications. It is available in a wide range of sizes, shapes, and corrugation profiles. It is available in helically produced pipe, deformed shapes (e.g., elliptical, pipe-arch, box, etc.), or structural plate sections. The structural plate sections are comprised of curved corrugated plates which are erected in the field via a bolted connection between adjacent plates. A typical structural plate pipe showing seam configurations is shown in Figure 1.

The corrugated profile produces a structural section with a high moment of inertia with relatively low cross-sectional area. With the corrugated cross-section, corrugated steel pipes have relatively low bending stiffness, and relatively high circumferential stiffness. This is in comparison with concrete pipe which has high bending stiffness and high circumferential stiffness, and thermoplastic pipe which has low bending stiffness and low circumferential stiffness.

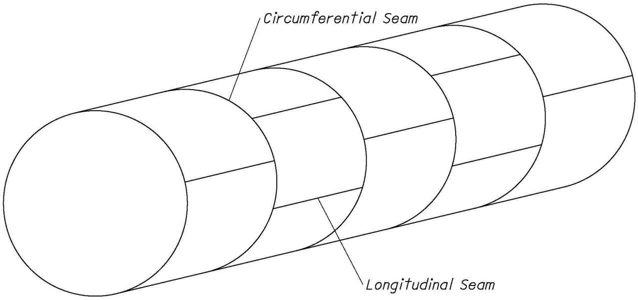

Figure 1 Structural plate pipe showing typical seam configuration

It is a common misconception to think of the installed pipe independent of the surrounding embedment soil. This ignores the complex interaction between the pipe and soil, the so-called soil-structure interaction phenomenon. When considering this interaction it is well known that the less stiff elements in the system will distribute loads to elements with greater stiffness. Defined by Selig, arching is the "redistribution of free-field stresses over a buried structure as a result of the difference in the deformation properties of the structure and the soil" [HYPERLINK \l "Sel75" *1*]. This phenomenon is called the arching effect. Positive soil arching occurs when a portion of the soil column (also known as the column load) above a pipe is transferred from the pipe to the surrounding soil. A negative arch occurs when the pipe attracts a loading greater than the soil column above the pipe. Arching can be measured in two ways. The first is the ratio of the normal soil pressure about the pipe to the geostatic stress or column load; the second is the ratio of the wall thrust, or circumferential stress to the wall stress generated by the geostatic stress.

A novel concept for corrugated structural plate pipe is to provide the bolted connections between the plates with an elongated bolt hole. This allows for the plates to "slip" over each other. The resultant circumferential shortening of the pipe results in a positive arching effect which results in less normal pressure on the pipe and thus less wall thrust. Currently, the load relief of this slotted joint pipe is accepted. However, the design of the slotted joint pipe relies on rule of thumb load relief estimates or the use of time consuming finite element software.

Slotted joint corrugated steel pipe is fabricated from structural plate with a 152.4 mm by 50.8 mm corrugation profile. The plates are then punched with a slotted bolt hole in lieu of the standard bolt hole. A typical slotted bolt-hole configuration is shown in Figure 2. Each of the mating plates is provided with a one- 12.7 mm slot in the circumferential direction, thus providing a total slip of

25.4 mm for each joint. The slot allows for a predetermined slip length when the pipe is placed under a soil load. The slotted joint corrugated steel pipe responds as a typical corrugated steel pipe up to the stress necessary to overcome the frictional stress of the bolted connection, called the slipping stress. After the slipping stress is exceeded the joint begins to slip. The slip results in circumferential shortening of the pipe. The soil responds by forming a soil arch and redistributing soil pressure, thus allowing for a greater soil load than without the slipping. After the joint slips the full length of the joint, the pipe then responds as a typical corrugated steel pipe.

Katona and Alk were the primary investigators of the theoretical aspects of the impact of slotted joints on the response of corrugated steel pipe. Through a series of laboratory tests on sections of slotted joint corrugated steel plates they developed a stress-strain model for the response of corrugated steel pipe with slotted joints. A typical load deformation diagram is shown in Figure 3. The model is comprised of four piecewise linear sections including elastic, slipping, post-slipping, and failure sections. The theoretical stress-strain model is shown in Figure 4. The theoretical elastic modulus of the various zones was calculated from load-deformation tests conducted on slotted joint plate samples [2].

The authors utilized these results to develop a series of non-dimensional ratios for elastic modulus and pipe yield stress for the various sections of the stress-strain model. They were then able to develop methods for calculating the response of a slotted joint corrugated steel pipe using both closed-form and finite element methods.

They accomplished this by utilizing an averaging technique to determine the slipping modulus of the entire circumference of the pipe wall by using an averaging technique based on the proportion of the pipe wall that was comprised of slotted joints to the proportion that was typical corrugated steel plate. The resulting equation is:

$$E^* = \frac{E_j}{(1 - J_r)\left(\frac{E_j}{E_e}\right) + J_r} \tag{1}$$

where E^* is the averaged elastic modulus of the pipe; E_j is the joint slipping modulus; E_e is the steel elastic modulus; and J_r is the ratio of total joint slip length to pipe circumference. The authors recommend a joint slipping modulus value of 0.0003 E_e and a post slipping modulus value of 0.0024 E_e [2]. However, in a later paper the authors recommend a post slipping modulus value of 0.5 E_e [3].

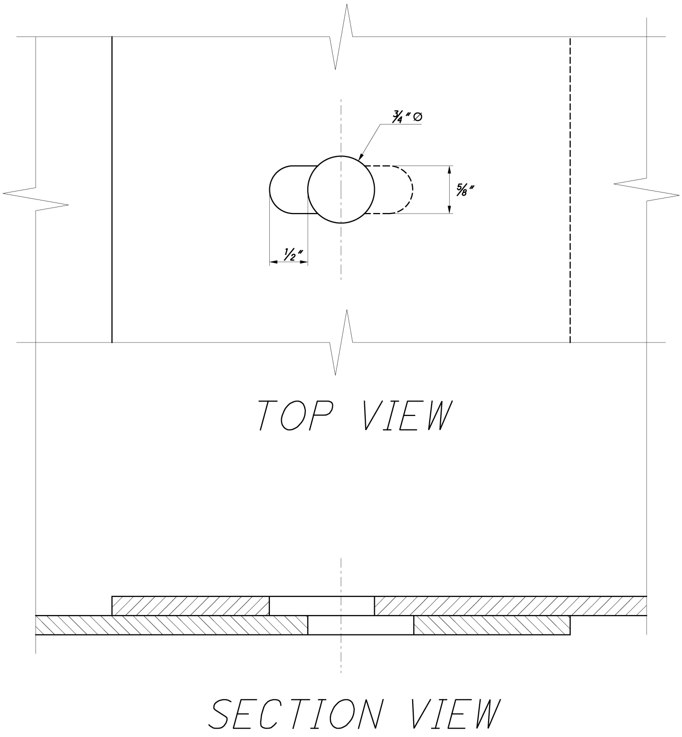

Figure 2 Typical slotted joint configuration

The closed form solution utilized the Burns and Richard elastic solution with an incremental loading scheme. The elastic modulus for the slipping and post-slipping zones were calculated based on the modulus of the joint and the modulus of the pipe using a weighted average of the relative circumferential length of each as discusses above [2].

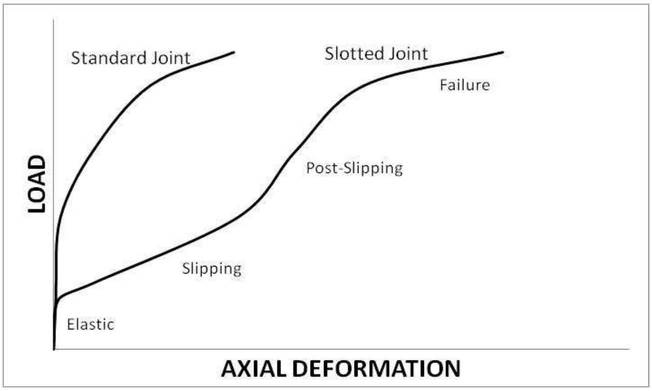

Figure 3 Typical slotted joint behaviour [2]

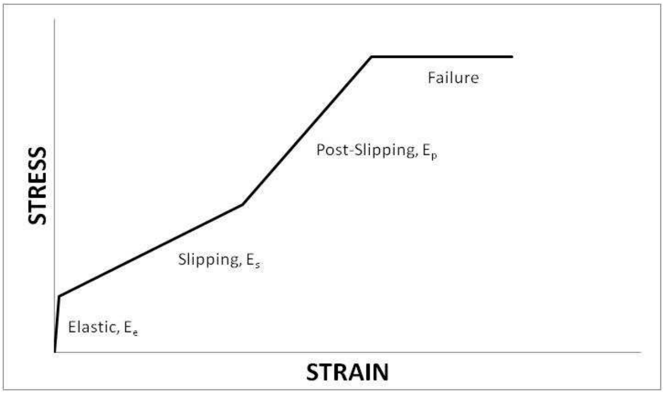

Figure 4 Theoretical model of slotted joint behavior [2]

The difficulty with this model is twofold; (1) the elastic solution of Burns and Richard is problematic because of the elastic assumption of both the shell and the continuum; (2) the method requires an incremental loading scheme, which for deeply buried corrugated steel pipe could result in significant computational effort.

The authors considered two differing methods for inclusion in the finite element method. The first method considered the slotted joint as a separate beam-column element with an elastic modulus equivalent to the slip zone elastic modulus experimentally derived. The other method utilized the previously described averaging method. The methods produced very similar results. The authors support the use of the averaging technique because of computational efficiency [2].

Lastly the authors compared the finite element results with the results of several instrumented culvert installation and found that the FEM produced reasonable results [2]. The slotted joint methodology has since been incorporated in the CANDE finite element software. Chang et al. also found that the CANDE slotted joint methodology produces results comparable to field installations [4].

Several researchers have published results of full-scale tests of structural plate corrugated steel pipe with slotted joints [5], [6].

Reduction in vertical soil pressure at the crown of the pipe, positive soil arching, was reported. The magnitude of the reduction in vertical soil pressure at the pipe crown varied from approximately 25 per cent to as much as 60 per cent of the free field stress.

2. EXISTING WORK COMPLETED AT OHIO UNIVERSITY

In 2001 a 6.40 m (twenty-one foot) corrugated structural plate pipe was installed in Meigs County Ohio under approximately 75 feet of earth cover. Pressure cells were installed about the periphery of the pipe and pressure readings were recorded during backfill placement. Complete details about the experimental installation and subsequent FEM modeling can be found in the literature. Important conclusions drawn by the authors include (1) that even though the structural plate pipe was not of the slotted joint design, joint slippage was visually observed; and (2) significant positive arching was measured in the field. Joint slipping was believed to have occurred using three distinct mechanisms; (1) plate slipping derived from the bolt holes being punched 1.6 mm greater in diameter than the bolt; (2) bolt rotation; and (3) plate rotation [7]. These observations became the impetus for testing of corrugated structural plate sections at the Ohio University laboratory.

Expanding on the field work in Meigs County Ohio completed by Ohio University, laboratory investigations of several sections of corrugated structural plates were conducted. A total of three corrugated steel plate joints were tested in a compression apparatus. The purpose of the experiments were to determine if the slipping of standard corrugated steel plates visually observed in the Meigs County installation could be recreated in the laboratory. Joint slippage was observed in each of the three tests. The magnitude of joint slippage was great enough to account for the load relief experienced by the Meigs County culvert. Full details of the experiment can be found elsewhere [8].

3. FINITE ELEMENT TEST CASES

The analysis of a typical pipe involves the determination of several design parameters. These include site geometry parameters such as the type and size of the pipe, the height of soil cover over the pipe, and geometry of the various soil zones. Soil zones that must be considered include the in situ soil zones, any embankment construction, and lastly the structural backfill surrounding the pipe.

The work conducted for this project was not indicative of any particular installation. Thus, precise discretization of field geometry was not required. General design parameters were varied to assess the general response of the installed pipe and surrounding soil to changes in pipe diameter, pipe wall thickness, installation type (embankment or trench), quality of the structural backfill, and length of joint slip for slotted joint structural plate pipe. The study included the analysis of 108 test cases. The study utilized two pipe diameters, 2.13 m diameter and 6.40 m diameter; two corrugated plate thicknesses, 4.8 mm and 6.3 mm; three backfill qualities of well graded granular material of 85, 90 and 95 per cent standard proctor density; five lengths of joint travel varying from no slip to 12.7 mm of slip; and two installation conditions, embankment and trench installation.

The public domain, pipe specific CANDE-2007 finite element software was utilized for the analysis of the test cases. Two finite element meshes were developed for analysis of the test cases. Separate meshes were constructed for the 2.13 m diameter pipe and for the 6.40 m diameter pipe. For both pipe diameters vertical symmetry was imposed. The general mesh configuration for the 2.13 m diameter pipe is shown in Figure 5.

The mesh boundaries were set based on multipliers of the pipe diameter. The lower boundary was set to one diameter below the invert of the pipe. The side boundary and top boundary were set to 1.5 times the pipe diameter. These boundary limits were selected to ensure that the boundaries would not influence the response of the pipe or the adjacent backfill. These limits are consistent with the boundary geometry utilized by the Level 2 CANDE meshes [9].

Construction of the pipe in the field is a stepwise procedure with backfill being placed and compacted along the side of the pipe in several construction lifts. CANDE allows for this procedure via the load step command and provides for a more realistic finite element model. The model for the 2.13 m pipe utilized a load step height of one foot to the top of the pipe. The model for the 6.40 m pipe utilized a load step height of three feet to the top of the pipe. Above the top of the pipe, for both test cases, the load step height increased as the distance from the top of the pipe increased.

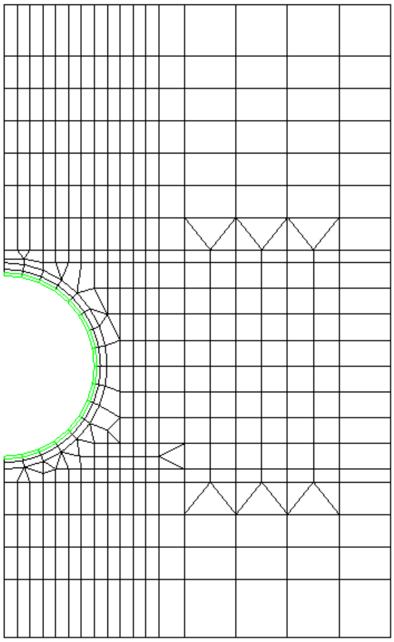

Figure 5 CANDE FEM mesh of the 2.13 m pipe

Sixteen load steps were utilized to completely develop the pipe meshes for each pipe diameter. Additional soil cover was then modeled as equivalent uniform nodal forces applied along the upper boundary in fourteen load steps for the 2.13 m pipe and ten load steps for the 6.40 m pipe.

The pipe was modeled using a series of connected beam-column elements. The 2.13 m diameter pipe utilized sixteen beam-column elements while the 6.40 m diameter pipe utilized fifteen beam-column elements. The corrugated steel plates were modeled using the CANDE bilinear stress strain material definition. The upper region of the bilinear stress strain model was set to an arbitrary low value of 68.95MPa. Additional material properties and section properties were taken from ASTM A796 for each of the two plate gages used in the analyses. The material and section properties are provided in Table 1. All of the beam-column elements were included with the first load step.

A total of four slotted joints were using in modeling the pipe. Thus, two joints were used in the FEM half-space. The slotted joints were placed at 45 degrees above and below the horizontal springline of the pipe. A stress of 34.13MPa was used as the onset of joint slipping. The ratio of slipping modulus to pipe modulus was set to 0. The ratio of post-slipping modulus to pipe modulus was set at 0.5.

Table 1 Material and section properties of corrugated steel plate

Modulus of Elasticity	200 GPa			
Material Yield	227.53 MPa			
Sheet Thickness (mm)	Area (mm^2/mm)	Moment of Inertia (mm^4/mm)	Section Modulus (mm^3/mm)	Radius of Gyration (mm)
4.8	5.8	1769.8	63.9	17.5
6.3	7.7	2392.5	83.9	17.6

The soil continuum consists of 398 elements for the 2.13 m diameter pipe and 399 elements for 6.40 m pipe. A total of five soil zones were defined for the test cases. Figure 6 shows the various soil zones for an embankment installation.

The in situ material, for both the trench and embankment installation, was modeled using the linear elastic soil model with a unit weight of 1.92 tonnes/m^3. The material was assigned an arbitrary elastic modulus of 6.9 MPa and a Poisson's Ratio of 0.3. The remaining soil zones were modeled using the Duncan/Selig hyperbolic bulk modulus model that is built into the CANDE model. CANDE also provides a series of parameters for the Duncan/Selig model that are consistent with laboratory tests for various soil types and backfill densities. These "canned" parameters were utilized for analysis of the test cases [9].

The bedding material was modeled using the Duncan/Selig soil model using the SW90 material. The unit weight of the material was set to 1.92 tonnes/m^3. Construction of a pipe often results in the haunch of the pipe being poorly compacted. The haunch area is inaccessible to compaction equipment. The material in this area was modeled as SW80 material to best represent the inability to completely compact this area. The overburden soil was modeled using the ML90 material with a unit weight of 1.92 tonnes/m^3. The structural backfill region was varied for the test cases. While the material type was consistently gravelly sand (SW material), three different soil densities were utilized. In an effort to assess the impact of soil quality on the installed pipe, backfill density was set at 85 per cent, 90 per cent, and 95 per cent of Standard Proctor Density. This essentially equates to a poor installation, an adequate installation, and a good installation.

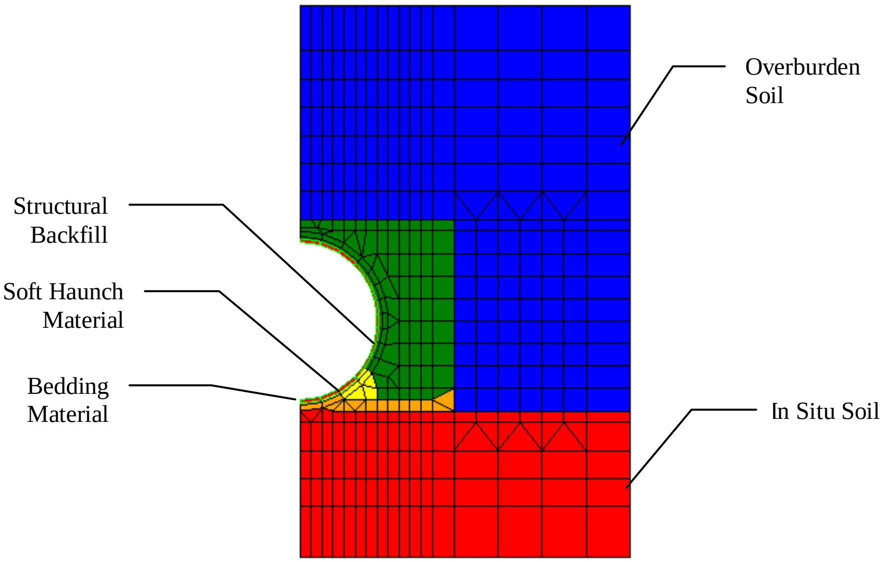

Figure 6 CANDE soil zones for an embankment installation.

Interface elements were used to connect the beam-column elements to the continuum elements. Interface elements are zero dimension elements that are used to permit relative movement between the two elements. The interface elements were modeled using a 0.3 frictional coefficient and a 1.75 kN/m tensile contact breaking force.

Boundary conditions were established for all four external mesh surfaces. The nodes along the lower mesh boundary were constrained as pin connections. The nodes along both vertical mesh boundaries were constrained as roller connections where movement in the horizontal direction was constrained.

The top surface of the mesh included nodal force vectors to simulate additional overburden pressure. For the 2.13 m diameter pipe, each load step, 17 through 34 were comprised of nodal forces equivalent to a uniform pressure of 51.4 kPa or approximately 2.74 m of overburden. For the 6.40 m diameter pipe, each load step, 16 through 23 were comprised of nodal forces equivalent to a uniform pressure of 34.5 kPa or approximately 6 feet of overburden.

4. FINITE ELEMENT RESULTS

Data compiled from the one hundred eight CANDE FEM test cases were analyzed. Of particular importance was the thrust in the pipe and the reaction of the soil at the crown of the pipe.

Numerical non-convergence of the FEM model can occur for a variety of causes. This is true for the CANDE FEM software; and other authors have reported numerical convergence difficulties [4], [7]. This issue typically results from the use of CANDE interface elements. However, it is also possible that the CANDE incremental loading can result in a load step too large. This results in non-convergence of the Duncan/Selig soil model. Non-convergence can also result from excessive stress in, or deformation of, the pipe.

A total of 108 test cases were analyzed. Convergence was not achieved for twelve of the test cases. Ten of the cases utilized 2.13 m pipe and two of the cases were 6.40 m pipe. Interface element non-convergence occurred ten times, while beam element non-convergence occurred twice.

Vertical crown soil pressure is an important measure of the response of the pipe and surrounding structural backfill. It is one of the two methods commonly used for estimating the vertical arching effect.

For the various non-zero slip lengths, vertical soil pressure at the crown of the pipe at maximum applied load varied from 741 kPa to 1434 kPa for the 3 gage, 2.13 m pipe and from 617 kPa to 1071 kPa for the 7 gage, 2.13 m pipe. The vertical soil pressure at the crown of the pipe at maximum applied load varied from 259 kPa to 421 kPa for the 3 gage, 6.40 m pipe and from 252 kPa to 336 kPa for the 7 gage, 6.40 m pipe.

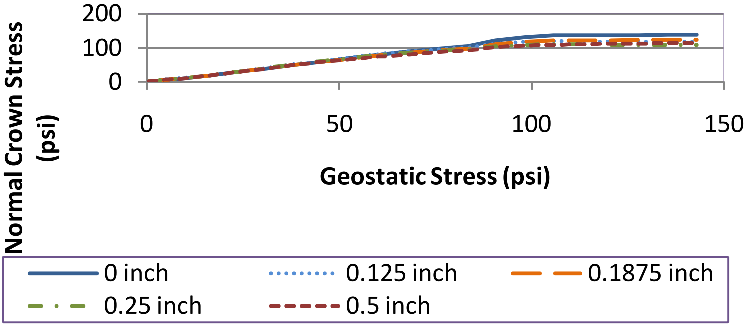

Figure 7 Vertical crown soil pressure versus applied stress for five slotted joint lengths

The vertical soil pressure at the crown of the pipe was graphed for each of the test cases. The data was grouped by pipe diameter, sheet thickness, and installation type. Figure 7 shows the results of the 2.13 m diameter pipe with a 3 gage wall thickness, installed in an embankment installation with good backfill. Similar graphs were produced for all of the test cases but are omitted for brevity.

As discussed previously, soil arching can be calculated as the portion of the geostatic stress that acts normal to the crown of the pipe. For each test case the vertical arching factor was calculated based on the ratio of the vertical crown soil pressure to the vertical crown soil pressure for the no slip case. The results for an embankment installation are tabulated in Table 2.

Table 2 Vertical arching based on vertical crown soil pressure-embankment installation.

Structural Backfill Compactive Effort	**Pipe Diameter and Gage**	**Joint Slip (mm)**			
		3.2	**4.8**	**6.4**	**12.7**
Good	2.13-3	0.853	0.892	0.773	0.824
	2.13-7	0.810	0.781	0.756	0.708
	6.40-3	0.988	0.960	0.937	0.876
	6.40-7	0.984	0.977	0.956	0.900
Medium	2.13-3	0.983	0.950	0.918	0.815
	2.13-7	0.917	0.881	0.851	0.789
	6.40-3	0.879	0.983	N/A	0.937
	6.40-7	1.007	0.959	0.986	0.940
Poor	2.13-3	1.093	0.877	N/A	0.983
	2.13-7	N/A	N/A	N/A	N/A
	6.40-3	0.809	0.794	0.835	0.799
	6.40-7	0.928	0.981	1.002	0.866

N/A=model nonconvergence

The pipe wall thrust stress is utilized in two of the design checks for structural plate corrugated steel pipe. Both the wall yielding and the seam strength checks utilize thrust stress as a primary input. Thus an accurate understanding of the thrust response of the slotted joint corrugated steel pipe is necessary.

A total of one hundred eight finite element test cases were analyzed. As discussed previously, numerical convergence could not be obtained for twelve of the test cases. Results for the thrust response of the remaining ninety-six test cases are presented herein.

For all of the test cases the maximum thrust occurred at the springline of the pipe. The results presented are for the maximum wall thrust.

For the various non-zero slip lengths, maximum thrust stress at the pipe springline due to maximum applied load varied from 99.9 kPa to 139.2 kPa for the 3 gage, 2.13 m pipe and from 106 kPa to 149.2 kPa for the 7 gage, 2.13 m pipe. The maximum thrust stress at the springline of the pipe due to the maximum applied load varied from 137.7 kPa to 172.1 kPa for the 3 gage, 6.40 m pipe and from 178.4 kPa to 212.6 kPa for the 7 gage, 6.40 m pipe.

The thrust stress at the springline of the pipe was graphed for each of the test cases. The data was grouped by pipe diameter, sheet thickness, and installation type. Figure 8 shows the results of the 2.13 m diameter pipe with a 3 gage wall thickness, installed in an embankment installation with good backfill. Similar graphs were produced for all of the test cases but are omitted for brevity.

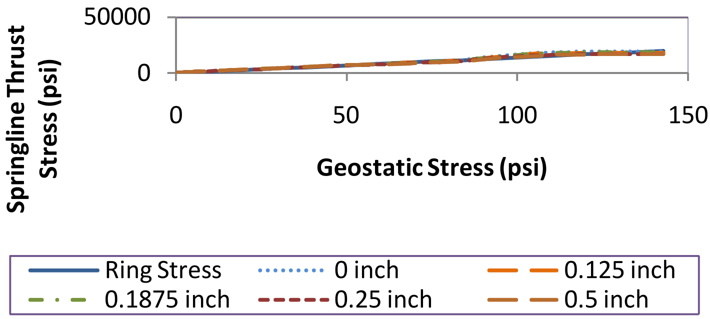

Figure 8 Springline thrust stress versus geostatic stress for five slotted joint lengths

Table 3 Vertical arching based on thrust stress-embankment installation

Structural Backfill Compactive Effort	Pipe Diameter and Gage	Joint Slip (mm)			
		3.2	4.8	6.4	12.7
Good	2.13-3	0.920	0.950	0.887	0.867
	2.13-7	0.965	0.946	0.933	0.870
	6.40-3	0.991	0.991	0.994	0.956
	6.40-7	0.986	0.982	0.975	0.953
Medium	2.13-3	0.998	0.992	0.987	0.967
	2.13-7	1.000	0.996	0.991	0.976
	6.40-3	1.000	0.982	N/A	0.971
	6.40-7	0.994	0.914	0.988	0.984
Poor	2.13-3	0.996	1.016	N/A	0.986
	2.13-7	N/A	N/A	N/A	N/A
	6.40-3	0.975	0.965	0.969	0.977
	6.40-7	0.978	0.931	1.010	0.972

N/A=model nonconvergence

In addition to the crown pressure measurement of soil arching, soil arching can also be calculated using wall thrust. For the case of the slotted joint corrugated structural plate pipe, the soil arching can be defined as the ratio of the wall thrust for the slip joint case to the wall thrust for the no slip case. The results for the embankment installation are tabulated in Table 3.

5. DEVELOPMENT OF SIMLIFIED DESIGN PROTCOL

Prior to the development of the simplified procedure for the design of slotted joint corrugated steel structural plate pipe, a brief review of the current design methodology is warranted.

The current design methodology is a four step procedure for deeply buried pipes. The fundamental basis for the design is the ring compression theory of White and Layer [10]. This is used for the determination of the proportion of the geostatic stress that is transferred to the pipe. The soil load results in ring stresses in the pipe wall. The White and Layer ring compression theory assumes a vertical arching factor of one. Thus the ring thrust is equivalent to one half of the applied geostatic load. The method also checks to ensure the pipe wall will yield in compression before the wall will buckle. The seam strength of the bolted plates are checked to ensure that the bolted seams in corrugated steel structural plate pipe have sufficient strength to transfer the wall thrust across the joint. Lastly, an empirical flexibility factor is utilized to ensure the pipe has sufficient rigidity to resist the forces applied to the pipe during transport and installation [9], [12]. Corrugated steel pipe design also requires checks for buckling and installation stiffness. Buckling resistance is a function of the diameter of the pipe and the radius of gyration of the pipe wall. There is no reason to believe that either of these properties is drastically changed when using a slotted joint pipe. Handling strength is an empirical equation based on pipe diameter and the elastic modulus and moment of inertia of the pipe wall. It is a measure of the ability of the pipe to withstand handling and installation forces. Again, there is no reason to believe that the handling strength of the pipe is significantly affected by the slotted joints.

The focus the simplified design method is on determining the proportion of the geostatic load that is imparted to the slotted joint steel corrugated structural plate pipe, and a check to ensure that the resulting wall thrust does not exceed the capacity of the pipe wall. A stepwise procedure will be utilized.

It is important to note that the validity of the ring compression theory of White and Layer is assumed. The redistribution of stresses arising from the slotted joint may result in bending moments in the wall of the pipe which are greater than the no slip case and invalidate ring compression theory. Additional analyses of the results are necessary to confirm the suitability of the theory.

The method presented herein utilizes two equations from the literature. The first is equation (1) used to determine the slipping modulus of the pipe soil system. The second, developed by McGrath, is equation (2) and is used to calculate the load on a pipe based on the combined effects of the soil stiffness and the ring stiffness of the pipe [13].

$$VAF = 0.76 - 0.71 \frac{(S_H - 1.17)}{(S_H + 2.92)} \tag{2}$$

where *VAF* is the vertical arching factor and S_H is the hoop stiffness factor defined as:

$$S_H = \frac{M_s}{EA/_R} \tag{3}$$

where M_s is the constrained soil modulus; *E* is the modulus of elasticity of the pipe (*E** in equation (1)); *A* is the wall area of the pipe; and *R* is the pipe radius [13].

The suitability of equation (2) was tested by plotting the hoop stiffness factor, S_H, of the pipe soil system versus the vertical arching factor determined from the CANDE analyses. The analyses only considered the stresses during joint slipping. The soil modulus was set to 34.5 MPa, 17.2 MPa, and 8.3 MPa for the good, medium, and poor backfill, respectfully. Figure 9 shows the results. The correlation of the linear trend line is quite good with an R^2=0.74. The general trend is evident.

The proposed design method utilizes equation (2) for the determination of the proportion of the geostatic stress that is transferred to the pipe. Reference is made to the various elastic modulus zones shown in Figure 4. For a particular slotted joint pipe, the ring stiffness and resultant vertical arching factor vary with the elastic modulus. All other properties remain constant.

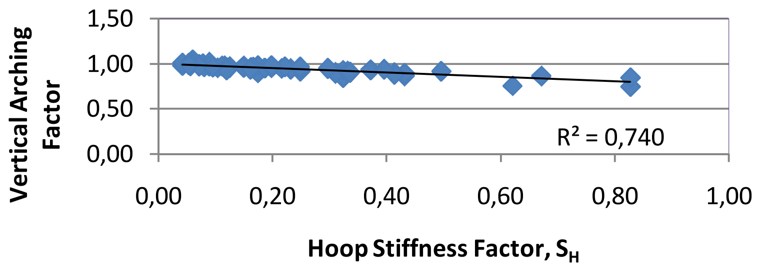

Figure 9 Vertical arching factor versus pipe soil ring stiffness

Up to the point of the onset of joint slipping, typically taken as 34.5 MPa, standard ring compression theory can be used to determine the pipe thrust stress. After this thrust stress is achieved, the joint will slip. As such, the rate of increase of the load on the pipe will be reduced. The reduction can be calculated using equation (2) using the pipe slipping modulus calculated using equation (1). A recommended value for the joint slipping modulus, E_j, is 0.0003 E_e [2]. The load reduction is then calculated as the ratio of the vertical arching factor for the slotted joint case to the vertical arching factor for the no slip case. This provides a relative measure of the load relief to be expected during joint slipping.

Next it is necessary to determine the stress level at the point of full joint closure, defined as σ_c. This is achieved by decoupling the slipping strain from the pipe wall strain. It is computationally convienient to ignore the pipe wall stain and only consider the slipping strain. Also, the response of the pipe wall during joint slipping is dominated by the slipping strain. The slipping strain at the point of joint closure is the joint closure ratio, J_r, from equation (1). The thrust stress at this point is then determined using the slipping strain and the pipe slipping modulus, $E*$.

Calculate Maximum Height of Cover for Slip Joint and No-Slip Cases

Conduit Diameter	$D := 252in = 6.401\,m$
Wall area	$Area := 0.304\,\frac{in^2}{in} = 7.722\cdot\frac{mm^2}{mm}$
Conduit Modulus	$E_p := 200GPa$
Conduit Yield Stress	$f_y := 228MPa$
Number of Joints	$Jt := 12$
Slipping Length	$Sl := 1in = 25.4\cdot mm$
Slipping Stress	$\sigma_s := 34.5MPa$
Unit Weight of Soil	$\gamma := 120\,\frac{lb}{ft^3} = 1.922\cdot\frac{tonne}{m^3}$
Modulus of Soil	$M_s := 34.5MPa$
Gravity Acceleration	$g = 9.807\,\frac{m}{s^2}$

Calculate Height of Cover at Onset of Joint Slip

$$H_s := \frac{2\sigma_s \cdot Area}{D \cdot \gamma \cdot g} \qquad H_s = 4.416\,m$$

Calculate Conduit Slipping Modulus

$$E_j := 0.0003E_p \qquad E_j = 60 \cdot MPa$$

$$J_r := \frac{Sl \cdot Jt}{\pi \cdot D} \qquad J_r = 0.0152$$

$$E_{star} := \frac{E_j}{(1 - J_r)\left(\frac{E_j}{E_p}\right) + J_r} \qquad E_{star} = 3.88 \cdot GPa$$

Calculate Vertical Arching Factor

$$S_{Hslip} := \frac{M_s}{\left(\frac{E_{star} \cdot Area}{\frac{D}{2}}\right)} \qquad S_{Hslip} = 3.683$$

$$S_H := \frac{M_s}{\left(\frac{E_p \cdot Area}{\frac{D}{2}}\right)} \qquad S_H = 0.071$$

$$VAF_{slip} := 0.76 - 0.71 \cdot \frac{(S_{Hslip} - 1.17)}{(S_{Hslip} + 2.92)} \qquad VAF_{slip} = 0.49$$

$$VAF_p := 0.76 - 0.71 \cdot \frac{(S_H - 1.17)}{(S_H + 2.92)} \qquad VAF_p = 1.021$$

$$VAF := \frac{VAF_{slip}}{VAF_p} \qquad VAF = 0.48$$

Calculate Thrust Stress at Joint Closure

$\Delta\sigma := E_{star} \cdot J_r$ $\Delta\sigma = 58.853 \cdot MPa$

$\sigma_c := \sigma_s + \Delta\sigma$ $\sigma_c = 93.353 \cdot MPa$

Calculate Height of Cover at Joint Clousure

$$H_c := \frac{2 \cdot \Delta\sigma \cdot Area}{D \cdot \gamma \cdot g} \cdot \frac{1}{VAF} + H_s \qquad H_c = 20.113\,m$$

Calculate Height of Cover at Wall Failure

$\Delta f_y := f_y - \sigma_c$ $\Delta f_y = 134.647 \cdot MPa$

$$H_f := \frac{2 \cdot \Delta f_y \cdot Area}{D \cdot \gamma \cdot g} + H_c \qquad H_f = 37.347\,m$$

Calculate Maximum Height of Cover for No Slip Case

$$H_{nc} := \frac{2 \cdot f_y \cdot Area}{D \cdot \gamma \cdot g} \qquad H_{nc} = 29.182 \cdot m$$

$$\%Gain := 1 - \frac{H_{nc}}{H_f} \qquad \%Gain = 21.863 \cdot \%$$

The pipe wall strain is a function of the elastic modulus of the steel material and the load on the pipe ignoring the load relief, or in other words the ring compression stress. Thus the apparent stress at the point of joint closure is the joint closure stress multiplied by the inverse of the vertical arching factor.

Beyond the point of joint closure the elastic modulus does not return to the pre-slip value of E_e. A recommended value for the post slipping modulus is 0.5 E_e [3]. However, it is conservative to set the post slipping elastic modulus equivalent to E_e and this is computationally more efficient. A parametric study of the post slipping vertical arching factor shows that assuming the post slipping modulus is equivalent to the material's elastic modulus results in a maximum error of approximately 12 per cent for the range of typical values used in the design of corrugated steel structural plate structures. It follows that ring compression theory is applicable for the post slipping analysis.

Sample calculations for a 6.40 m diameter , 3-gage conduit, with 12 - 25.4 mm slots, installed utilizing good backfill proceed as follows.

In this case there is a 21.9 per cent gain in the maximum height of cover for the slotted joint pipe.

It is important to note that the above examples only consider thrust stress. A complete design would also include checks for seam strength, wall buckling, and installation stiffness.

Next a comparison is made between the arching factors calculated using the simplified solution and the finite element solution. A similar installation is the best that can be accomplished because the finite element model uses a hyperbolic soil model while the simplified solution assumes a linear elastic soil. The CANDE finite element solution for thrust stress at the maximum applied pressure is considered. The maximum applied stress was 0.985 MPa for the 2.31 m pipe and 0.422 MPa for the 6.40 m. Using a soil with a unit weight of 1.92 tonnes/m^3, this is equivalent to 22.40 m and 52.24 m of earth cover for the 2.314 m and 6.40 m pipe, respectively.

The arching factor using the CANDE software was calculated as the ratio of the maximum thrust stress for the slotted joint case to the thrust stress for the no-slip case. The arching factor for the simplified solution is taken as the ratio of the thrust stress for the slotted joint case to the thrust stress for the ring compression case. Table 4 summarizes the results of the calculations for a select data set. A positive difference indicates the simplified solution over-predicts the ring stress while a negative difference indicates that the simplified solution under-predicts the ring stress. The maximum difference between the CANDE solution and the simplified solution is 7.56 per cent and the minimum difference is -0.16 per cent.

The CANDE data set considered was for an embankment installation with good backfill. This data set was utilized because special construction procedures and a higher level of installation quality control are typically implemented when using slotted joint pipes. A high quality installation is most likely to occur.

Table 4 Comparison of soil arching factors for CANDE and the simplified solution.

	CANDE solution Joint slip (mm)			Simplified solution Joint slip (mm)			% Difference Joint slip (mm)		
	3.2	6.4	12.7	3.2	6.4	12.7	3.2	6.4	12.7
2.13-3	0.920	0.887	0.867	0.982	0.960	0.918	6.33%	7.56%	5.56%
2.13-7	0.965	0.933	0.870	0.982	0.961	0.919	1.71%	2.88%	5.39%
6.40-3	0.991	0.994	0.956	0.989	0.974	0.942	-0.16%	-2.10%	-1.50%
6.40-7	0.986	0.975	0.953	0.989	0.974	0.943	0.28%	-0.07%	-1.10%

5. CONCLUSIONS

Numerical investigations were performed to evaluate the performance of corrugated steel structural plate pipes with stress relieving slotted joints. The finite element computer software CANDE was utilized to model 108 tests cases. The study utilized two pipe diameters, 2.13 m diameter and 6.40 m diameter; two corrugated plate thicknesses, 4.8 mm and 6.3 mm; three backfill qualities of well graded granular material of 85, 90 and 95 per cent standard proctor density; and five lengths of joint travel varying from no slip to 12.7 mm of slip.

Non convergence of the CANDE model was an issue during this investigation. Model convergence could not be obtained for 10 of the 108 test cases. The most common cause of non convergence was non convergence of the interface elements.

When considering vertical soil arching as the change in vertical soil pressure at the crown of the pipe, all of the test cases except for five exhibited positive soil arching. The magnitude of the vertical arching varied from 1.095 to 0.809. When considering vertical arching as the change in the maximum thrust stress of the pipe, all of the test cases except for nine exhibited positive soil arching. The magnitude of the vertical arching varied from 1.02 to 0.830. The cause of the negative vertical soil arching is unknown and is attributed to the complex nature of the soil pipe system.

A simplified solution was developed for the design of slotted joint structural plate corrugated steel pipe. The method utilizes a reduced modulus of elasticity during joint slipping developed by Katona and Alk [2], [3]; and an equation for determining the vertical soil arching factor based on soil stiffness and pipe hoop stiffness developed by McGrath [13]. The simplified method produced results for the maximum thrust stress that are in good agreement with the CANDE finite element solution. The maximum difference between the simplified solution and the finite element solution was 7.56 per cent.

It is recommended that additional testing be conducted to ensure that the results are consistent with results from installed pipes. Additionally, the theory needs to be verified to ensure additional bending moments in the pipe wall that develop with the slotted joint pipe do not invalidate the ring compression theory.

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Streszczenie

Rury z blachy falistej wyposażone w złącza z otworami wydłużonymi pozwalają na swobodny przesuw śrub i nasuwanie się dopasowanych blach na siebie w określonym stopniu. Powodowane tym skracanie pierścienia ma kilka konsekwencji dla zainstalowanej rury. Główna korzyść polega na zmniejszeniu nacisku gruntu działającego na rurę, co w rezultacie zmniejsza siłę osiową w ścianie rury.

W tym referacie przedstawiono reakcje zainstalowanego złącza z otworem wydłużonym oraz nośnej stalowej rury z blachy falistej z wykorzystaniem metody elementów skończonych. Opracowano uproszczone rozwiązanie do projektowania nośnych stalowych rur ze stali falistej wyposażonych w wydłużone otwory na śruby i porównano je z rezultatami analizy elementów skończonych.

Słowa kluczowe: rura z blachy falistej, interakcja między konstrukcją a gruntem, przepust ze złączami z otworami wydłużonymi